HANDOUT 4 – MULTIVARIATE REGRESSION (II) + DUMMY VARIABLES

AGENDA

- California School Districts with four covariates
- Reading regression tables
- Introduction to dummy variables
- Defining dummy variables
- Using dummy variables
- Program evaluation
- More than two categories
- Vocabulary
- Takeaways

BIBLIOGRAPHY FOR TODAY´S CLASS

- Goldin (2017) How to win the battle of sexes over pay.
- Stock and Watson (2007): 1.2, 3.4, 5.3, 9.1, 9.4 (**)
- Angrist and Pischke (2014): 2 (pp. 39-67)

CALIFORNIA SCHOOL DISTRICTS WITH FOUR COVARIATES

Let us add to the equation we used to estimate the impact of class size (Student-teacher ratio) on test scores income, computers per student, and fraction of students that have English as a second language:

Y = *testscr*	district's mean math/reading 5th grade test score
$\mathbf{X_1}$ = *str*	district's student-teacher ratio (class size)
$\mathbf{X_2}$ = *avginc*	district's mean family income (in 000's of $)
$\mathbf{X_3}$ = *comp_stu*	computers per student
$\mathbf{X_4}$ = *el_pct*	fraction of students with English as second language

```
. regress testscr str avginc comp_stu el_pct, robust

Linear regression                                     Number of obs =     420
------------------------------------------------------------------------------
             |               Robust
     testscr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         str |    .052132   .3016771     0.17   0.863    -.5408737    .6451377
      avginc |    1.48448   .0923195    16.08   0.000     1.303008    1.665952
    comp_stu |    13.8179   9.161063     1.51   0.132    -4.189974    31.82577
      el_pct |  -.4795127   .0285596   -16.79   0.000    -.5356522   -.4233733
       _cons |   636.0783   6.707339    94.83   0.000     622.8937    649.2629
------------------------------------------------------------------------------
```

What is the SRF estimated by this model?

What is the predicted test score for a district with student-teacher ratio=20, average income=$40,000, computers per student=0.5, and fraction of students that have English as a second language=20%?

Interpret the student-teacher ratio coefficient? Does it have the expected sign? Is it important from a practical standpoint? Is it statistically significant?

READING REGRESSION TABLES

- Most research articles display regression results in nice tables, not Stata!
- In the typical table, each column contains a single regression, with explanatory variables listed at left. This allows easy comparison of different regression specifications
- Coefficients are reported, usually with standard errors below in parentheses so that you can divide to compute a t-statistic (sometimes they display the t-statistic directly)
- Statistically significant coefficients are often noted by stars, as specified in table notes. A typical example: *p<.05,**p<.01 (but it might change, you need to read table notes)

That table for all the regressions we have analyzed in class would look like this:

Dependent var.: Test score	(1)	(2)	(3)
Student-teacher ratio	-2.280**	-0.649	0.0521
	(0.519)	(0.353)	(0.302)
Average income		1.839**	1.484**
		(0.115)	(0.092)
Computers per student			13.82
			(9.161)
Fraction English learners			-0.480**
			(0.029)
Constant	698.9**	638.7**	636.1**
	(10.36)	(7.301)	(6.707)
N	420	420	420

INTRODUCTION TO DUMMY VARIABLES

Our regression models have used only continuous variables, which take a range of values (test scores, class sizes, income).

We will often be interested in variables indicating whether an observation falls into a particular category. Is the individual:

- Male or female?
- Of indigenous origins?
- Foreign worker?
- Urban or rural?

Dummy variables divide data into categories.

Researchers often have to assume simple categories for research purposes (two genders, four racial/ethnic groups, etc.).

DEFINING DUMMY VARIABLES

Dummy variables take only the values zero or one, and indicate whether an observation belongs to a category (0/1 are arbitrary values but make regressions easier to interpret)

Dummy variable
Binary variable
Indicator variable

For example, we can define female as:

- *Female* = 1 if individual is female
- *Female* = 0 if individual is male

Alternatively, we can define male as:

- *Male* = 1 if individual is male
- *Male* = 0 if individual is female

Here is data from 2009 American Community Survey MA residents 30-50 years old with positive earnings:

- *Income:* earnings in 2008 (in 000s dollars)
- *Age:* age in years
- *Educ:* years of schooling completed
- *Male:* =1 if male (=0 otherwise)
- *Female:* =1 if female (=0 otherwise)

Person	Income	Age	Educ	Male	Female
1	50	38	16	0	1
2	62	38	13	1	0
3	45	47	16	0	1
4	25	33	17	0	1
	.	.	.	.	.
1432	73	43	17	1	0

What are the characteristic of person 3?

What does male+female equal?

When a set of variables adds to a fixed number, we can call those variables **multicollinear** or **collinear**. **Multicollinearity** causes the dummy variable trap.

USING DUMMY VARIABLES

We can also do this with a bivariate regression, using this PRF:

$$\widehat{income}_i = \beta_0 + \beta_1 female_i + \varepsilon_i$$

the SRF for predicted earnings is thus:

$$\widehat{income}_i = \hat{\beta}_0 + \hat{\beta}_1 female_i$$

given this PRF and SRF, what are the predicted earnings of:

- Men?

- Women?

- What is the interpretation of $\hat{\beta}_0$?

- What is the interpretation of $\hat{\beta}_1$?

Now look at the regression results:

```
. regress income female, robust noheader
```

income	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-22.38741	2.30123	-9.73	0.000	-26.90156	-17.87326
_cons	66.75743	1.927088	34.64	0.000	62.97721	70.53766

- What is the difference between women and men?

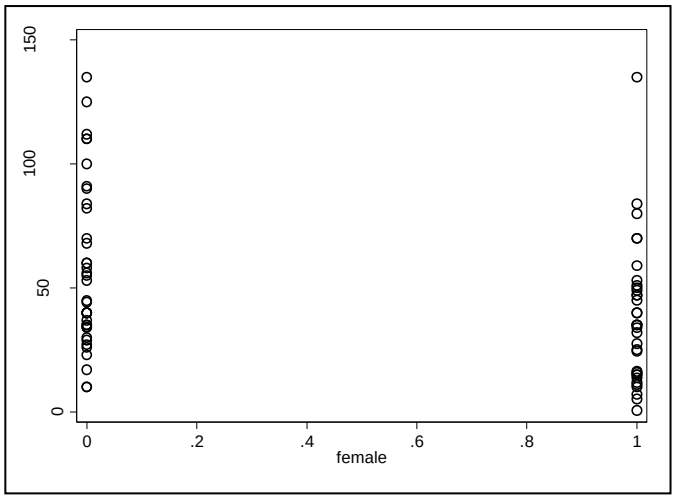
- What it the sample average income for men?

- What is the sample average income for women?

- What is the t-statistics for the test of the null hypothesis of no income difference in this population?

- Should we reject of fail to reject that null hypothesis?

Here is a scatter plot to visualize this dummy variable regression:



A bivariate regression with a dummy variable can be used for a comparison-of-means test between groups.

Interpreting dummy variable coefficients requires knowing **the omitted group** against which comparison are made.

We used men as the omitted group and compared women to them.

If we had instead used this PRF, our results would look like:

$$income_i = \alpha_0 + \alpha_1 male_i + \varepsilon_i$$

```
. regress income male, robust noheader
```

income	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
male	22.38741	2.30123	9.73	0.000	17.87326	26.90156
_cons	44.37003	1.257773	35.28	0.000	41.90275	46.8373

What if we include both dummy variables in the regression?

$$income_i = \beta_0 + \beta_1 female_i + \beta_2 male_i + \varepsilon_i$$

```
. regress income female male, robust noheader
note: male omitted because of collinearity
```

income	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-22.38741	2.30123	-9.73	0.000	-26.90156	-17.87326
male	0	(omitted)				
_cons	66.75743	1.927088	34.64	0.000	62.97721	70.53766

…that would lead to the **dummy variable trap**.

The trap is driven by our inclusion of at least once collinear variable, a variable that can be expressed as a linear combination of other variables.

Mind the omitted group in the interpretation of the coefficients.

PROGRAM EVALUATION

Randomized experiments assign individuals to two categories, a treatment group and a control group. Bivariate regression with a dummy variable can thus estimate the impact of the treatment:

- Define a dummy variable *treat* equal to 1 for those in treatment group and 0 for those in control group.
- Write out the PRF: $Y_i = \beta_0 + \beta_1 treat_i + \varepsilon_1$
- Estimate β_1, the outcome different between the two groups.

If you reject the null hypothesis H_0: $\beta_0 = 0$, we have found evidence of different outcomes between the two groups.

Because treatment is randomly assigned, no other factors should differ on average between the two groups. $\hat{\beta}_1$ will therefore give a causal estimate of the effect of the treatment on the outcome.

We previously used a comparison-of-means test to estimate the impact of the Tennessee STAR experiment on class size reduction.

- Students randomly assigned to a small class scores 13.7 points higher than students assigned to a regular class.
- We reject the null hypothesis that the two groups had equal test scores (t-statistics of 6.38).

Let's re-do this analysis with a bivariate regression approach.

Define a dummy variable small, equal to 1 for those assigned to a small class and 0 for those assigned to a regular class.

Then run: $testscr_i = \beta_0 + \beta_1 small_i + \varepsilon_i$

```
. regress testscr small, robust noheader
```

testscr	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
small	13.74055	2.154628	6.38	0.000	9.516677	17.96443
_cons	918.2013	1.135073	808.94	0.000	915.9762	920.4265

- Interpret $\hat{\beta}_0$

- Interpret $\hat{\beta}_1$

- Is the difference between the groups statistically significant?

MORE THAN TWO CATEGORIES

We can use dummy variables with more than two categories.
For example, do different racial/ethnic groups in Massachusetts have different earnings?
We will create one dummy variable for each of three groups (assume each person must be in exactly one of these categories):

White: =1 if white (=0 otherwise)
Black: =1 if black (=0 otherwise)
Hispanic: =1 if Hispanic (=0 otherwise)

Person	Income	Age	Educ	Male	Female	White	Black	Hispanic
1	50	38	16	0	1	1	0	0
2	62	38	13	1	0	1	0	0
3	45	47	16	0	1	0	0	1
4	25	33	17	0	1	0	1	0
	.	.	.	.	.	.	.	.
1432	73	43	17	1	0	1	0	0

```
. regress income black hispanic, robust noheader
```

income	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
black	-15.03508	3.993734	-3.76	0.000	-22.86929	-7.200867
hispanic	-24.87962	2.46249	-10.10	0.000	-29.7101	-20.04914
_cons	57.05987	1.263512	45.16	0.000	54.58133	59.53841

- Interpret $\hat{\beta}_0$

- Interpret $\hat{\beta}_1$, $\hat{\beta}_2$

- Why not include a variable race, equal to 1 for whites, 2 for blacks, and 3 for Hispanics?

TAKEAWAYS

- Dummy variables take only the values 0 and 1: They are used to describe data that fall into 2 or more categories.
- Inclusion of variables that can be expressed as linear combination of other variables already in the regression, leads to the dummy trap.
- Dummy variables can be used to estimate treatment effects in randomized experiments, comparing outcomes in treatment (dummy=1) and control groups (zero).
- The key to interpreting dummy variables is to be clear about which is the omitted category that the group of interest is being compared to.

VOCABULARY

- Dummy variable/ Binary variable/ Indicator variable: a variable that is either 0 or 1. A binary variable is used to indicate a binary outcome.
- Coefficient multiplying the dummy variable (as opposed to slope): measure the average difference between the groups coded with value 1 and 0 (the default or base group).
- Multicollinearity/ Multicollinear variables/ Collinear variables: occurs when one of the regressors is an exact linear function of the regressors.
- Dummy variables trap: a problem caused by including a full set of binary variables in a regression together with a constant regressor (intercept), leading to perfect multicollinearity.
- Omitted group: the dummy group omitted represents the reference group, and the differential intercept coefficients for the dummy groups included in the model are differential values with reference to the value for this omitted group, that is, the intercept coefficient of the constant term.

OPTIONAL (MULTIVARIATE REGRESSION II)

- The default formula used for computing standard errors of OLS regression coefficients assumes that the data are **homoskedastic** (variance around the regression line is the same for all values of the predictor variable X)
- Most data is at least somewhat **heteroskedastic** (variance around the regression line varies by value of the predictor variable X)
- The "robust" option for standard errors alters the formula to account for this.