Economic Growth with Bubbles

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Abstract

We develop a stylized model of economic growth with bubbles. In this model, changes in investor sentiment lead to the appearance and collapse of macroeconomic bubbles or pyramid schemes. We show how these bubbles mitigate the effects of financial frictions. During bubbly episodes, unproductive investors demand bubbles while productive investors supply them. These transfers of resources improve the efficiency at which the economy operates, expanding consumption, the capital stock and output. When bubbly episodes end, these transfers stop and consumption, the capital stock and output contract. We characterize the stochastic equilibria of the model and argue that they provide a natural way of introducing shocks to asset prices into business cycle models.

JEL classification: E32, E44, O40

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1 Introduction

Modern economies often experience episodes of large movements in asset prices that cannot be explained by changes in economic conditions or fundamentals. It is commonplace to refer to these episodes as bubbles popping up and bursting. Typically, these bubbles are unpredictable and generate substantial macroeconomic effects. Consumption, the capital stock and output all tend to surge when a bubble pops up, and then collapse or stagnate when the bubble bursts. Here, we address the following questions: What is the origin of these bubbly episodes? Why are they unpredictable? How do bubbles affect consumption, the capital stock and output? In a nutshell, the goal of this paper is to develop a stylized view or model of economic growth with bubbles.

The theory presented here features two idealized asset classes: productive assets or "capital" and pyramid schemes or "bubbles". Both assets are used as a store of value or savings vehicle, but they have different characteristics. Capital is costly to produce but it is then useful in production. Bubbles play no role in production, but initiating them is costless.[1] We consider environments with rational, informed and risk neutral investors that hold only those assets that offer the highest expected return. The theoretical challenge is to identify situations in which these investors optimally choose to hold bubbles in their portfolios and then characterize the macroeconomic consequences of their choice.

Our research builds on the seminal papers of Samuelson (1958) and Tirole (1985) who viewed bubbles as a remedy to the problem of dynamic inefficiency.[2] Their argument is based on the dual role of capital as a productive asset and a store of value. To satisfy the need for a store of value, economies sometimes accumulate so much capital that the investment required to sustain it exceeds the income that it produces. This investment is inefficient and lowers the resources available for consumption. In this situation, bubbles can be both attractive to investors and feasible from a macroeconomic perspective. For instance, a pyramid scheme that absorbs all inefficient investments in each period is feasible and its return exceeds that of the investments it replaces.

The Samuelson-Tirole model provides an elegant and powerful framework to think about bub-

[1] It is difficult to find these idealized asset classes in financial markets, of course, as existing assets bundle or package together capital and bubbles. Yet much can be learned by working with these basic assets. To provide an obvious analogy, we have surely learned much by studying theoretical economies with a full set of Arrow-Debreu securities even though only a few bundles or packages of these basic securities are traded in the real world.

[2] Our research is also indebted to previous work on bubbles and economic growth. Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993) extend the Samuelson-Tirole model to economies with endogenous growth due to externalities in capital accumulation. In their models, bubbles slow down the growth rate of the economy. Olivier (2000) uses a similar model to show how, if tied to R&D firms, bubbles might foster technological progress and growth.

bles. However, the picture that emerges from this theory is hard to reconcile with historical evidence. First, the model features deterministic bubbles that exist from the very beginning of time and never burst. This is contrary to the observation that in real economies bubbles pop up and burst. We therefore need a model in which bubbles are transient, that is, a model of bubbly episodes. Second, and most importantly, in the Samuelson-Tirole model bubbles raise consumption by reducing inefficient investments. As a result, bubbles slow down capital accumulation and lower output. In the real world, bubbly episodes tend to be associated with consumption booms indeed. But they also tend to be associated with expansions in the capital stock and output. A successful model of bubbles must come to grips with these correlations.

We build such a model by introducing investor sentiment shocks and imperfect financial markets into the theory of rational bubbles. Since bubbles have no intrinsic value, their current size depends on market expectations regarding their future size, i.e. on "investor sentiment". Introducing shocks to investor sentiment is therefore crucial to generate realistic bubble dynamics in the model.[3] Introducing financial frictions is also crucial because these create rate-of-return differentials and allow efficient and inefficient investments to coexist. Our key observation is then quite simple: bubbles not only reduce inefficient investments, but also increase efficient ones. In our model, bubbly episodes are booms in consumption *and* efficient investments financed by a reduction in inefficient investments. If efficient investments increase enough, bubbly episodes expand the capital stock and output. This turns out to be the case under a wide range of parameter values.[4]

To understand these effects of bubbly episodes, it is useful to analyze the set of transfers that bubbles implement. Remember that a bubble is nothing but a pyramid scheme by which the buyer surrenders resources today expecting that future buyers will surrender resources to him/her. The economy enters each period with an initial distribution of bubble owners. Some of these owners bought their bubbles in earlier periods, while others just created them. When the market for bubbles opens, on the demand side we find investors who cannot obtain a return to investment above that of bubbles; while on the supply side we find consumers and investors who can obtain

[3] To the best of our knowledge, Weil (1987) was the first to consider stochastic bubbles in general equilibrium.

[4] The introduction of financial frictions also solves an empirical problem of the theory of rational bubbles. Abel et al. (1989) examined a group of developed economies and found that, in all of them, investment falls short of capital income. This finding, which means that the average investment is dynamically efficient, has often been used to argue that in real economies the conditions for the existence of rational bubbles are not satisfied. But this argument is not quite right. Even if the average investment is dynamically efficient, the economy might contain some dynamically inefficient investments that could support a bubble. Moreover, it is also possible that an expansionary bubble, by lowering the return to investment, creates itself the dynamically inefficient investments that support it. Woodford (1990) and Azariadis and Smith (1993) were, to the best of our knowledge, the first to show that financial frictions could relax the conditions for the existence of rational bubbles.

a return to investment above that of bubbles. When the market for bubbles clears, resources have been transferred from inefficient investors to consumers and efficient investors.

A key aspect of the theory is how the distribution of bubble owners is determined. As in the Samuelson-Tirole model, our economy is populated by overlapping generations that live for two periods. The young invest and the old consume. The economy enters each period with two types of bubble owners: the old who acquired bubbles during their youth, and the young who are lucky enough to create new bubbles. Since the old only consume, bubble creation by efficient young investors plays a crucial role in our model: it allows them to finance additional investment by selling bubbles.

There has been quite a bit of interest recently on the effects of bubbles in the presence of financial frictions: (i) Caballero and Krishnamurthy (2006) and Farhi and Tirole (2011) show that bubbles can be a useful source of liquidity;[5] (ii) Kocherlakota (2009) and Martin and Ventura (2011) show that bubbles can also raise collateral or net worth; and (iii) Ventura (2011) shows that bubbles can lower the cost of capital.[6] Unlike these papers, and due to the simplicity of our setup, we are able to provide a full characterization of all the stochastic equilibria of the model and show that they provide a natural way of introducing asset-price shocks into business-cycle models. Finally, there are two papers that have used rational bubbles to interpret recent macroeconomic developments: Kraay and Ventura (2007) use a model of bubbles and capital flows to study the origin and effects of global imbalances, while Martin and Ventura (2011) use a model of bubbles and the financial accelerator to interpret the 2007-08 financial crisis and its effects.

Stylized facts:

The theory developed here is motivated by two stylized facts: (i) modern economies experience large movements in asset prices that seem unrelated to economic fundamentals; and (ii) these asset price movements tend to generate substantial macroeconomic effects.

That there are large movements in asset prices is clear in the data. For instance, a study of industrial countries by the International Monetary Fund, IMF (2003), found that equity price busts occurred on average once every 13 years whereas housing busts occurred on average every 20 years.

[5] There is, of course, a long tradition of papers that view fiat money as a bubble. Indeed, Samuelson (1958) adopted this interpretation. For a recent paper that also emphasizes the liquidity-enhancing role of fiat money in the presence of financial frictions, see Kiyotaki and Moore (2008).

[6] This paper is the closest in spirit to ours. Ventura (2011) models a multi-country world in which financial frictions impede capital flows. In this model, there are many markets for country bubbles. When this market is active, the capital stock falls in the country, but this lowers the price of investment goods and raises the capital stock in the rest of the world. The paper then uses a few examples to study how shocks are transmitted across countries.

Both equity and housing price busts entailed significant price declines, on average of 45 and 30 percent respectively. That some of these movements in asset prices are unrelated to fundamentals seems to be the case as well. Take, for instance, the case of the US stock price boom and bust of the late 1990s. From 1994 to its peak in 2000, the Dow Jones Industrial Average more than tripled, from 3,600 to 11,722.98. LeRoy (2004) showed that this increase in the value of equity far exceeded the growth of GDP or of corporate earnings. He also found scant evidence for other "fundamental" explanations, most notably those based on demographics and on the valuation of intangible capital.[7] Another case in point is the more recent boom and bust in real home prices in many industrialized economies. In the US, for example, real home prices increased by 85% between 1997 and 2006. This increase, which was spectacular by historical standards, led Shiller (2005) to analyze explanations based on fundamentals, such as demographic trends, the rise of construction costs and the evolution of interest rates. He concluded that these "fundamentals" were unlikely to lie behind the increase in home prices, which have since then contracted by 35%.[8]

There is ample evidence that equity and housing price changes in industrialized economies are closely correlated with – and tend to lead – output growth.[9] Of the asset price busts of the postwar period analyzed in IMF (2003), for example, the average equity bust was associated with GDP losses of about 4 percent whereas the average housing bust was associated with GDP losses of about 8 percent. Going beyond simple correlations, there is mounting evidence that points to a direct and independent effect of asset prices on investment decisions. Gan (2007), for instance, analyzed firm- and loan-level data corresponding to the late 1990s in Japan in order to quantify the impact of a large decline in asset markets on firms' investment decisions. Based on a sample containing all publicly traded manufacturing firms, he found that the collapse of land prices had a significant and negative effect on corporate investment.[10] More recently, Chaney et al. (2008) have documented similar results for the US economy using firm-level data for the 1993-2007 period.[11]

[7] Something similar can be said regarding the boom in the japanese stock market during the late 1980s. Between 1987 and 1990, the Tokyo Stock Price Index nearly doubled. This boom was followed by a bust, and by 1993 the increase in stock prices had been completely undone. As LeRoy did for the case of the United States, French and Poterba (1991) analyzed the evolution of japanese stock prices during this period and concluded that they were unlikely to be explained by fundamentals.

[8] Once again, there is a similar story for the case of Japan, in which land prices nearly tripled in the second half of the 1980s. At its peak in 1990, the market value of all the land in Japan famously exceeded four times the land value of the United States. This boom was followed by a bust in land prices, which nearly halved between 1990 and 1993. French and Poterba (1991) also found that these developments were hard to attribute to fundamentals.

[9] See IMF (2000) for a review of the literature that documents these correlations.

[10] Specifically, he found a reduction in the investment rate of 0.8% for every 10% decline in land value. In a related study, Goyal and Yamada (2004) found that the evolution of stock prices in Japan during the late 1980s and early 1990s also had a significant effect on corporate investment.

[11] In particular, they found that a one dollar increase in the value of its real estate leads the average US corporation

2 The Model

This section develops a model that builds on the seminal contributions of Samuelson (1958), Diamond (1965) and Tirole (1985). It introduces two new elements which turn out to be crucial for the analysis. The first one is random creation and destruction of bubbles. The second one is financial frictions. None of these two pieces is new. But their combination creates a novel and quite suggestive view of the origins and effects of bubbly episodes in real economies.

2.1 Setup

Consider an economy inhabited by overlapping generations of young and old. Time starts at $t = 0$ and then goes on forever. Each generation contains a continuum of individuals of size one, indexed by $i \in I_t$. Individuals maximize expected old-age consumption, i.e. $U_{it} = E_t \{c_{it+1}\}$; where U_{it} and c_{it+1} are the utility and the old-age consumption of individual i of generation t. Individuals supply one unit of labor when young. Since they care only about old age consumption, they save their entire labor income. The only choice these individuals make is how to allocate their savings between capital and bubbles. Since individuals are risk-neutral, they choose the portfolio that maximizes the expected return to their savings.

The output of the economy is given by a Cobb-Douglas production function: $F(l_t, k_t) = l_t^{1-\alpha} \cdot k_t^{\alpha}$ with $\alpha \in (0,1)$, where l_t and k_t are the labor force and capital stock, respectively. Since the young have one unit of labor, $l_t = 1$. Markets are competitive and factors of production are paid the value of their marginal product:

$$w_t = (1-\alpha) \cdot k_t^{\alpha} \quad \text{and} \quad r_t = \alpha \cdot k_t^{\alpha-1}, \tag{1}$$

where w_t and r_t are the wage and the rental rate, respectively.

The stock of capital in period $t+1$ depends on the investment made by generation t during its youth.[12] In particular, we have that:

$$k_{t+1} = A_t \cdot s_t \cdot k_t^{\alpha}, \tag{2}$$

to raise its investment by 6 cents. This implies that a drop in real estate prices of 35%, like the one that has happened in the US since 2006, depresses aggregate investment by more than 5% purely because of financial frictions.

[12] We assume that (i) capital fully depreciates in production; and (ii) the first generation found some positive amount of capital to work with, i.e. $k_0 > 0$.

where s_t is the investment rate, i.e. the fraction of output that is devoted to capital formation; and A_t is the investment efficiency, i.e. the units of capital that are created for each unit of output that is devoted to capital formation.

To solve the model, we need to find the investment rate and its efficiency. At this point, it is customary to assume that the young use all their savings to build capital. This means that the investment rate equals the savings of the young. Since the latter equal labor income, which is a constant fraction $1 - \alpha$ of output, the investment rate is constant as in the classic Solow model:

$$s_t = 1 - \alpha \equiv s. \tag{3}$$

How efficient are the young at building capital? We now introduce a key feature of the model, namely, a friction in financial markets that lowers investment efficiency. We start by creating gains from financial trade by assuming that some individuals are better at investing than others. In particular, a fraction $\varepsilon \in [0, 1]$ of the young can produce one unit of capital with one unit of output, while the rest only have access to an inferior technology that produces $\delta < 1$ units of capital with one unit of output. We refer to these two types as "productive" and "unproductive" investors, respectively. If financial markets worked well, unproductive investors would lend their resources to productive ones and these would invest on everyone's behalf. The aggregate investment efficiency would be one. We assume however that this is not possible because of some unspecified market imperfection. As a result, unproductive investors are forced to make their own investments. This means that the average investment efficiency is given by:

$$A_t = \varepsilon + (1 - \varepsilon) \cdot \delta \equiv A. \tag{4}$$

Since all individuals invest the same amount, the average efficiency of investment is determined by the population weights of both types of investors.

Substituting Equations (3) and (4) into Equation (2), we find the dynamics of the capital stock:

$$k_{t+1} = A \cdot s \cdot k_t^{\alpha}. \tag{5}$$

Equation (2) constitutes a very stylized version of a workhorse model of modern macroeconomics. This model can be extended by adding more sophisticated formulations of preferences and technology, various types of shocks, a few market imperfections, and a role for money. Instead, we follow

Samuelson (1958) and Tirole (1985) and open a market for bubbles or pyramid schemes.

2.2 Equilibria with bubbles

We introduce now a market for bubbles. These are intrinsically useless assets, and the only reason to purchase them is to resell them later. Let b_t be the stock of old bubbles in period t, i.e. already existing before period t or created by earlier generations; and let b_t^N be the stock of new bubbles, i.e. added in period t or created by generation t. We assume that bubbles start randomly and without cost. This implies that new bubbles constitute a pure profit or rent for those that create them. We use the notation b_t^{NP} and b_t^{NU} to denote the stock of new bubbles created by productive and unproductive investors, respectively. Naturally, $b_t^{NP} + b_t^{NU} = b_t^N$. Finally, we assume that there is free disposal of bubbles. This implies that $b_t \geq 0$, $b_t^{NP} \geq 0$ and $b_t^{NU} \geq 0$.

Let us first describe how the market for bubbles works.[13] On the supply side, there are two types of bubble owners: the old who acquired bubbles during their youth and the young who are lucky enough to create new ones. On the demand side, there can only be the young since the old do not save. Taking this into account, we find that the following conditions must hold in all dates and states of nature:

$$E_t\left\{\frac{b_{t+1}}{b_t + b_t^{NP} + b_t^{NU}}\right\}\begin{cases} = \delta\cdot\alpha\cdot k_{t+1}^{\alpha-1} & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s\cdot k_t^{\alpha}} < 1 \\ \in\left[\delta\cdot\alpha\cdot k_{t+1}^{\alpha-1}, \alpha\cdot k_{t+1}^{\alpha-1}\right] & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s\cdot k_t^{\alpha}} = 1 \\ = \alpha\cdot k_{t+1}^{\alpha-1} & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s\cdot k_t^{\alpha}} > 1 \end{cases}, \tag{6}$$

$$0 \leq b_t \leq s\cdot k_t^{\alpha}. \tag{7}$$

Equation (6) follows from the first-order conditions of the portfolio allocation problem of individuals. For bubbles to be attractive to a particular investor, they must deliver at least the same return as capital. The return to holding the bubble consists of its growth over the holding period. The purchase price of the bubble is $b_t + b_t^{NP} + b_t^{NU}$, and the selling price is b_{t+1}. The return to investing

[13] Let b_{it} and b_{it}^N denote the bubble demanded and created by individual $i \in I_t$ in period t, respectively. We can write the intertemporal budget constraint of this individual as follows:

$$c_{it+1} = r_{t+1}\cdot A_i\cdot(w_t + b_{it}^N - b_{it}) + \left(\frac{b_{t+1}}{b_t + b_t^N}\right)\cdot b_{it},$$

where $A_i = 1$ if individual i is productive and $A_i = \delta$ otherwise and $\dfrac{b_{t+1}}{b_t + b_t^N}$ is the return to holding bubbles.

on capital equals the rental rate divided by the cost of capital, which is one for productive investors and δ^{-1} for unproductive ones. Equation (6) then recognizes that the marginal buyer of the bubble changes as the bubble grows. If the bubble is small, the marginal buyer is an unproductive investor and the expected return to the bubble must equal the return to unproductive investments. If the bubble is large, the marginal buyer is a productive investor and the expected return to the bubble must be the return to productive investments.[14] Equation (7) imposes the non-negativity constraints on both bubbles and capital. That bubbles must be positive follows from our free-disposal assumption. That bubbles cannot exceed the savings of the young, i.e. $s \cdot k_t^{\alpha} + b_t^N \geq b_t + b_t^N$, follows from the non-negativity constraint on the capital stock.

One can summarize this discussion by saying that the theory imposes two restrictions on the type of bubbles that can exist. On the one hand, bubbles must grow fast enough or otherwise the young will not be willing to purchase them. This restriction is embedded in Equation (6). On the other hand, the aggregate bubble cannot grow too fast or otherwise the young will not be able to purchase them. This restriction is embedded in Equation (7). The tension between these two restrictions is what determines the set of equilibrium bubbles, as we show in section 3.

The presence of a market for bubbles has potentially important macroeconomic effects that work through capital accumulation. To see this, we first derive the dynamics of the capital stock in the presence of bubbles:

$$k_{t+1} = \begin{cases} A \cdot s \cdot k_t^{\alpha} + (1-\delta) \cdot b_t^{NP} - \delta \cdot b_t & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha}} < 1 \\ s \cdot k_t^{\alpha} - b_t & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon) \cdot s \cdot k_t^{\alpha}} \geq 1 \end{cases}, \tag{8}$$

Equation (8) has two steps that depend on who is the marginal buyer of the bubble. When the bubble is small, the marginal buyer is an unproductive investor. In this case, capital accumulation equals the savings of the productive investors times their efficiency (which is one), i.e. $\varepsilon \cdot s \cdot k_t^{\alpha} + b_t^{NP}$; plus the savings of the unproductive investors minus the value of the bubbles they purchase times their efficiency (which is δ), i.e. $\delta \cdot \left[(1-\varepsilon) \cdot s \cdot k_t^{\alpha} + b_t^{NU} - b_t - b_t^{NP} - b_t^{NU}\right]$. When the bubble is large, the marginal buyer is a productive investor. In this case, unproductive investors do not build capital and capital accumulation equals the savings of the productive investors i.e. $\varepsilon \cdot s \cdot k_t^{\alpha} + b_t^{NP}$; minus the bubbles they purchase, i.e. $b_t + b_t^{NP} + b_t^{NU} - (1-\varepsilon) \cdot s \cdot k_t^{\alpha} - b_t^{NU}$.

[14] Bubbles cannot deliver a higher return than productive investments. Asssume this were the case. Then, nobody would invest and the return to investment would be infinite. But this means that the bubble would be growing at an infinite rate and this is not possible.

Equation (8) nicely illustrates the two macroeconomic effects of bubbles. The first one is the classic *crowding-out* effect: when the old sell bubbles to the young, consumption grows and investment falls. This is why b_t slows down capital accumulation. Interestingly, the bubble crowds out first unproductive investments. It is only when there are no unproductive investments left that the bubble starts to crowd out productive investments. This ability of the bubble to eliminate the 'right' investments raises average investment efficiency and minimizes this crowding-out effect. The second macroeconomic effect of bubbles is a new *reallocation* effect: when the productive young sell bubbles to the unproductive young, productive investments replace unproductive ones. This effect further raises average investment efficiency and explains why b_t^{NP} speeds up capital accumulation. The net effect of these two effects is unclear at this point since we do not know the relative size of b_t and b_t^{NP}. We return to this issue in section 3 when we discuss the equilibria of this economy.[15]

We now provide a formal definition of a competitive equilibrium for this economy. For a given initial capital stock and bubble, $k_0 > 0$ and $b_0 \geq 0$, a competitive equilibrium is a sequence $\left\{k_t, b_t, b_t^{NP}, b_t^{NU}\right\}_{t=0}^{\infty}$ satisfying Equations (6), (7) and (8). Closing down the market for bubbles is equivalent to adding the additional equilibrium restriction that $b_t = b_t^{NP} = b_t^{UP} = 0$ for all t. This restriction cannot be justified on 'a priori' grounds, but we note that there always exists one equilibrium in which it is satisfied. This "fundamental" equilibrium, as described by Equation (5), is the one macroeconomics has focused on almost exclusively.

Before analyzing the equilibria of this model, we explain how it differs from (and what it adds to) the original models of Samuelson (1958) and Tirole (1985). Unlike us, both Samuelson and Tirole restricted their analysis to the subset of equilibria that are deterministic and do not involve bubble creation or destruction. That is, they imposed the additional restrictions that $E_t b_{t+1} = b_{t+1}$ and $b_t^N = 0$ for all t. With these restrictions, any bubble must have existed from the very beginning of time and it can never burst, i.e. its value can never be zero. This makes their models unsuitable to study business cycles. We therefore relax these restrictions here and allow for stochastic equilibria with bubble creation. Unlike us, both Samuelson and Tirole assumed that financial markets are frictionless. Since this allows productive investors to invest on behalf of unproductive ones, this is akin to imposing the additional restriction that $\delta = 1$. With this restriction, bubbles only have

[15] For simplicity, we have assumed a financial friction that prevents financial intermediation altogether. Nothing substantial would change, however, if financial frictions constrained intermediation without eliminating it. As long as there is limited intermediation, at the equilibrium interest rate: (i) unproductive investors are forced to invest more than their desired amount; and (ii) productive investors are prevented from investing their desired amount. Hence, there are incentives for unproductive investors to demand bubbles and for productive investors to supply them, giving rise to the same two effects mentioned in the text.

crowding-out effects and slow down capital accumulation. This makes their models inconsistent with the empirical evidence that bubbly episodes tend to speed up capital accumulation. We therefore introduce financial frictions and allow for the possibility that bubbles be expansionary.[16]

3 Bubbly episodes and their macroeconomic effects

An important payoff of analyzing stochastic equilibria with bubble creation and destruction is that this allows us to rigorously capture the notion of a bubbly episode. Generically, the economy fluctuates between periods in which $b_t = b_t^N = 0$ and periods in which $b_t > 0$ and/or $b_t^N > 0$. We say that the economy is in the *fundamental state* if $b_t = b_t^N = 0$. We say instead that the economy is experiencing a *bubbly episode* if $b_t > 0$ and/or $b_t^N > 0$. A bubbly episode starts when the economy leaves the fundamental state and ends when the economy returns to the fundamental state. We study next the nature of bubbly episodes and their macroeconomic effects.

3.1 Existence of bubbly episodes

To study the types of bubbly episodes that can occur in equilibrium, it is useful to exploit a trick that makes the model recursive. Let x_t, x_t^{NP} and x_t^{NU} be the stock of old and new bubbles as a share of the savings of the young or wealth of the economy, i.e. $x_t \equiv \frac{b_t}{s \cdot k_t^{\alpha}}$, $x_t^{NP} \equiv \frac{b_t^{NP}}{s \cdot k_t^{\alpha}}$ and $x_t^{NU} \equiv \frac{b_t^{NU}}{s \cdot k_t^{\alpha}}$. Then, we can rewrite Equations (6) and (7) as saying that, if $x_t > 0$, then

$$E_t x_{t+1} \begin{cases} = \frac{\alpha}{s} \cdot \frac{\delta \cdot \left(x_t + x_t^{NP} + x_t^{NU}\right)}{A + (1-\delta) \cdot x_t^{NP} - \delta \cdot x_t} & \text{if } \frac{x_t + x_t^{NP}}{1-\varepsilon} < 1 \\ \in \left[\frac{\alpha}{s} \cdot \frac{\delta \cdot \left(x_t + x_t^{NP} + x_t^{NU}\right)}{A + (1-\delta) \cdot x_t^{NP} - \delta \cdot x_t}, \frac{\alpha}{s} \cdot \frac{x_t + x_t^{NU} + x_t^{NP}}{1 - x_t}\right] & \text{if } \frac{x_t + x_t^{NP}}{1-\varepsilon} = 1 \\ = \frac{\alpha}{s} \cdot \frac{x_t + x_t^{NP} + x_t^{NU}}{1 - x_t} & \text{if } \frac{x_t + x_t^{NP}}{1-\varepsilon} > 1 \end{cases}, \tag{9}$$

$$0 \leq x_t \leq 1. \tag{10}$$

Equations (9) and (10) fully describe the bubble dynamics that can take place in our economy. There are two sources of randomness in these dynamics: shocks to bubble creation, i.e. x_t^{NP} and x_t^{NU}; and shocks to the value of the existing bubble, i.e. x_t. Any admissible stochastic process

[16] There are other differences between our model and those of Samuelson and Tirole regarding preferences and technology that do not affect the results of the analysis.

for $\{x_t, x_t^{NP}, x_t^{NU}\}$ satisfying Equations (9) and (10) describes an equilibrium of the model. By admissible, we mean that this stochastic process must ensure that $x_t^{NP} \geq 0$ and $x_t^{NU} \geq 0$ for all t. Conversely, any equilibrium of the model can be expressed as an admissible stochastic sequence for $\{x_t, x_t^{NP}, x_t^{NU}\}$.

The following proposition provides the conditions for the existence of bubbly episodes:

Proposition 1 *Bubbly episodes are possible iff* $\alpha < \begin{cases} s \cdot \dfrac{A}{\delta} & \text{if } A > 1-\varepsilon \\ s \cdot \dfrac{A}{\delta} \cdot \max\left\{1, \dfrac{1}{4 \cdot (1-\varepsilon) \cdot A}\right\} & \text{if } A \leq 1-\varepsilon \end{cases}.$

To prove Proposition 1 we ask if, among all admissible stochastic processes for $\{x_t, x_t^{NP}, x_t^{NU}\}$ that satisfy Equation (9), there is at least one that also satisfies Equation (10). Consider first the case in which there is no bubble creation after a bubbly episode starts. Figure 1 plots $E_t x_{t+1}$ against x_t, using Equation (9) with $x_t^{NP} = x_{t_0}^{NP}$, $x_t^{NU} = x_{t_0}^{NU}$ and $x_t^{NP} = x_t^{NU} = 0$ for all $t > t_0$, where t_0 is the period in which the episode starts. The left panel shows the case in which $\alpha \geq s \cdot \dfrac{A}{\delta}$ and the slope of $E_t x_{t+1}$ at the origin is greater than or equal to one. This means that any initial bubble would be demanded only if it were expected to continuously grow as a share of labor income, i.e. if it violates Equation (10), and this can be ruled out. The right panel of Figure 4 shows the case in which $\alpha < s \cdot \dfrac{A}{\delta}$. Now the slope of $E_t x_{t+1}$ at the origin is less than one and, as a result, $E_t x_{t+1}$ must cross the 45 degree line once and only once. Let x^* be the value of x_t at that point. Any initial bubble $x_{t_0+1} > x^*$ can be ruled out. But any initial bubble $x_{t_0}^N \leq x^*$ can be part of an equilibrium since it is possible to find a stochastic process for x_t that satisfies Equations (9) and (10).

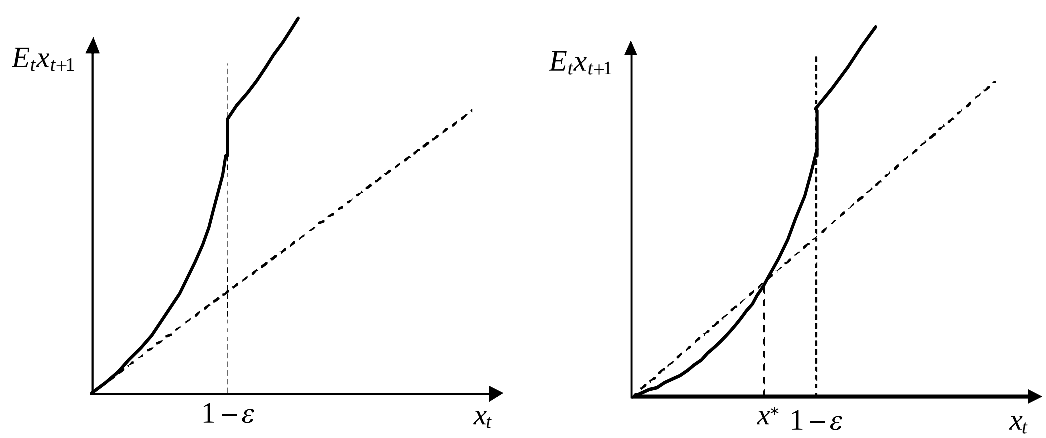

Figure 1

Is it possible that bubble creation relaxes the conditions for the existence of bubbly episodes? The answer is negative if we consider bubble creation by unproductive investors, i.e. x_t^{NU}. To see this, note that bubble creation shifts upwards the schedule $E_t x_{t+1}$ in Figure 1. The intuition is clear: new bubbles compete with old bubbles for the income of next period's young, reducing their return and making them less attractive.

Consider next bubble creation by productive investors, i.e. x_t^{NP}. Equation (9) shows that this type of bubble creation shifts the schedule $E_t x_{t+1}$ upwards if $x_t \in (0, A] \cup (1-\varepsilon, 1]$, but it shifts it downwards if $x_t \in (A, 1-\varepsilon]$. To understand this result, it is important to recognize the double role played by bubble creation by productive investors. On the one hand, new bubbles compete with old ones for the income of next period's young. This effect reduces the demand for old bubbles and shifts the schedule $E_t x_{t+1}$ upwards. On the other hand, productive investors sell new bubbles to unproductive investors and use the proceeds to invest, raising average investment efficiency and the income of next period's young. This effect increases the demand for old bubbles and shifts the schedule $E_t x_{t+1}$ downwards. This second effect operates whenever $x_t \leq 1-\varepsilon$, and it dominates the first effect only if $x_t \geq A$. Hence, if $A > 1-\varepsilon$, bubble creation by productive investors cannot relax the condition for the existence of bubbly episodes.

If $A \leq 1-\varepsilon$, bubble creation does relax the condition for the existence of bubbles. Namely, this condition becomes $\alpha < s \cdot \frac{A}{\delta} \cdot \max\left\{1, \frac{1}{4 \cdot (1-\varepsilon) \cdot A}\right\}$. Figure 2 provides some intuition for this result by plotting $E_t x_{t+1}$ against x_t, using Equation (9) and assuming that $x_t^{NU} = 0$ and

$$x_t^{NP} = \begin{cases} 0 & \text{if } x_t \in (0, A] \cup (1-\varepsilon, 1] \\ 1-\varepsilon-x_t & \text{if } x_t \in (A, 1-\varepsilon] \end{cases},$$

for all $t > t_0$. In both panels, this bubble creation by productive investors shifts the schedule $E_t x_{t+1}$ downward. The left panel shows the case in which this does not not affect the conditions for the existence of bubbly episodes, i.e. $4 \cdot (1-\varepsilon) \cdot A > 1$. The right panel shows instead the case in which bubble creation by productive investors weakens the conditions for the existence of bubbly episodes, i.e. $4 \cdot (1-\varepsilon) \cdot A < 1$. This completes the proof of Proposition 1.

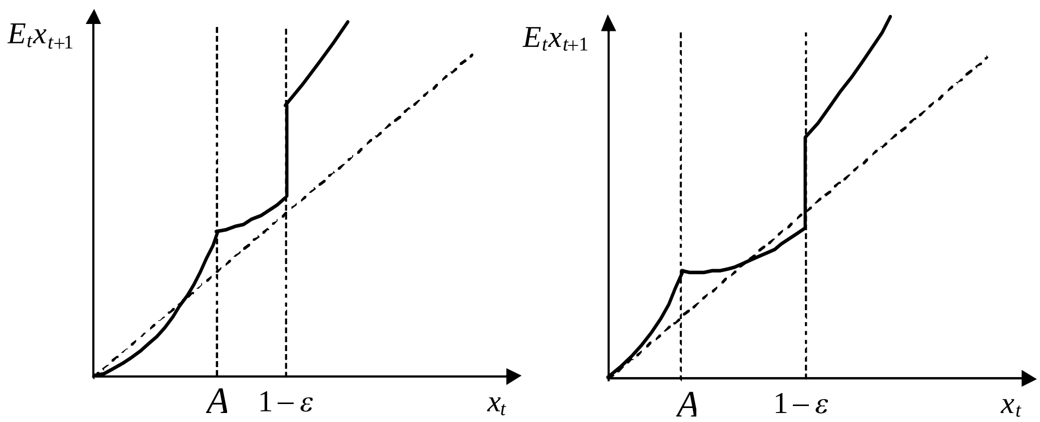

Figure 2

Proposition 1 provides the condition for existence of bubbly episodes of any sort. It is also useful to describe the conditions for the existence of bubbles according to their effects on capital accumulation. Recall that these effects depend on whether x_t^{NP} is smaller or greater than $x_t \cdot \dfrac{\delta}{1-\delta}$. We label a bubbly episode as contractionary if $x_t^{NP} < x_t \cdot \dfrac{\delta}{1-\delta}$ throughout its duration. We similarly label a bubbly episode as expansionary if $x_t^{NP} > x_t \cdot \dfrac{\delta}{1-\delta}$ throughout its duration.[17] With these definitions at hand, we can state the following proposition:

Proposition 2 *Contractionary bubbly episodes are possible iff* $\alpha < \alpha_C \equiv s \cdot \dfrac{A}{\delta}$. *Expansionary bubbly episodes are possible iff* $\alpha < \alpha_E \equiv s \cdot \dfrac{A}{\delta} \cdot \begin{cases} (1-\delta) & \text{if } A > 0.5 \\ \dfrac{1}{4 \cdot (1-\varepsilon) \cdot A} & \text{if } A \leq 0.5 \end{cases}$.

The proof of Proposition 2 follows almost immediately from the proof of Proposition 1 and we omit it here. Instead, we summarize the content of Proposition 2 with the help of Figure 3.[18]

[17] Formally, the existence of a contractionary (expansionary) bubbly episode requires the existence of an admissible stochastic process $\{x_t, x_t^{NP}, x_t^{NU}\}$ satisfying $E_t x_{t+1} < x_t$ and $x_t \cdot \dfrac{\delta}{1-\delta} < x_t^{NP}$ ($x_t \cdot \dfrac{\delta}{1-\delta} > x_t^{NP}$) for all t, where $E_t x_{t+1}$ is as in Equation (9). Some bubble episodes are neither contractionary nor expansionary according to these definitions since their effects on the capital stock and output vary through time or across states of nature.

[18] Figure 3 has been drawn for a fixed value of $\varepsilon < 0.5$. This guarantees that Region IV exists.

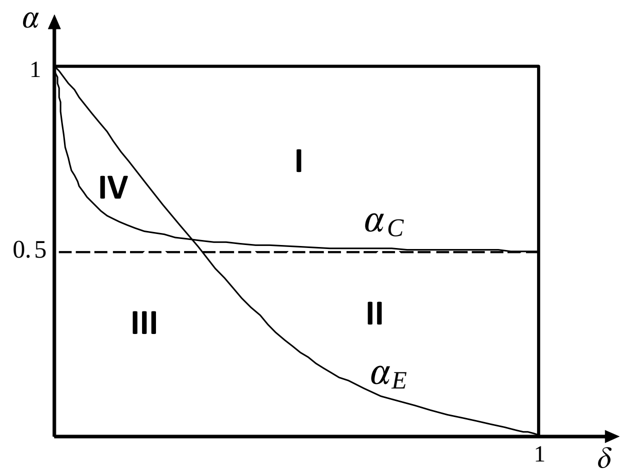

Figure 3

Bubbly episodes are possible in Regions II-IV, but not in Region I. In Regions II and III, $\alpha < \alpha_C$ and contractionary episodes are possible. In Region III and IV, $\alpha < \alpha_E$ and expansionary episodes are possible. In the limiting case $\delta \rightarrow 1$, only contractionary episodes are possible. As δ declines, the value of α required for the existence of both types of bubbly episodes declines. In the limiting case $\delta \rightarrow 0$, both types of bubbly episodes are possible regardless of α.

3.2 Bubbles and dynamic inefficiency

During a bubbly episode, the young reduce their investments and purchase bubbles. They do so voluntarily in the expectation that the revenues from selling these bubbles will exceed the foregone investment income. These revenues are nothing but the reduction in the investments of the next generation of young minus the value of any new bubbles. Thus, bubbly episodes are possible if there exists a chain of investments that is expected to absorb resources, that is, a chain whose cost is expected to exceed the income it produces in all periods. More formally, let I_t be a chain of investments and let D_t be the resources it absorbs. This chain is expected to absorb resources in all periods if, for all t,

$$E_t \{I_{t+1} - R_{t+1} \cdot I_t\} = E_t \{D_{t+1}\} \geq 0, \tag{11}$$

where R_{t+1} is the equilibrium return to the investments in the chain. We say that a chain is "dynamically inefficient" if it satisfies Equation (11). We provide next the economic intuition behind Propositions 1 and 2 by showing that bubbles can exist if the chains of investments they replace are dynamically inefficient *at the equilibrium return to investment.* Intuitively, investors are happy not to make these investments and instead purchase bubbles or pyramid schemes. The latter can offer as much as $E_t\{I_{t+1}\}$, while the investments can only offer $E_t\{R_{t+1} \cdot I_t\}$. We emphasize that Condition (11) must be evaluated at the equilibrium rate of return because this observation plays a subtle but crucial role in what follows: a chain of investments might not be dynamically inefficient in the fundamental or other equilibria, and yet this same chain might be dynamically inefficient in the equilibrium in which the bubble replaces it. It is the latter that is required for a bubbly episode to exist.

In the proof of Proposition 1, we began by considering bubbly episodes in which there is no bubble creation after their start. To determine whether these episodes are possible, we must simply check whether there exist dynamically inefficient chains of investments to be replaced, i.e. satisfying Condition (11) for some $D_t \geq 0$. Since it is easier to construct dynamically inefficient chains of unproductive investments, we take a chain of such investments $I_t = x_t \cdot s \cdot k_t^{\alpha}$. Since the equilibrium rate of return to unproductive investments is $R_t = \delta \cdot \alpha \cdot k_t^{\alpha-1}$, this chain satisfies Condition (11) if and only if

$$E_t x_{t+1} \cdot s \cdot k_{t+1}^{\alpha} \geq \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} \cdot x_t \cdot s \cdot k_t^{\alpha} = \frac{x_t \cdot \delta}{A - \delta \cdot x_t} \cdot \alpha \cdot k_{t+1}^{\alpha} \geq 0, \tag{12}$$

for all t.[19] The LHS of Condition (12) is $E_t\{I_{t+1}\}$ while the RHS is $E_t\{R_{t+1} \cdot I_t\}$. A chain of investments can satisfy Condition (12) if and only if

$$\alpha < s \cdot \frac{A}{\delta}.$$

Otherwise x_t would have to grow continuously and eventually exceed one, which is not possible. But this is the condition for the existence of bubbly episodes without bubble creation that we found in the proof of Proposition 1. Since these episodes are all contractionary, this is also the condition for being in regions II and III of Figure 3.

We then asked whether bubble creation could relax the conditions for bubbly episodes to exist. At first sight, one might be tempted to dismiss this possibility at once. With bubble creation, bubbles replace chains of investment that absorb a strictly positive amount of resources, i.e. $D_t > 0$; and this seems to make Condition (11) more stringent. But this reasoning is incomplete because it fails to recognize that Condition (11) must be evaluated at the equilibrium rate of return, which

[19] Here we have used Equation (8) and the definition of x_t to eliminate k_t.

might be lower in the equilibrium with bubble creation. Thus, take again a chain of unproductive investments $I_t = \left(x_t + x_t^{NP} + x_t^{NU}\right) \cdot s \cdot k_t^{\alpha}$ that absorbs resources $D_t = \left(x_t^{NP} + x_t^{NU}\right) \cdot s \cdot k_t^{\alpha}$. Since the equilibrium rate of return to unproductive investments is $R_t = \delta \cdot \alpha \cdot k_t^{\alpha-1}$, this chain satisfies Condition (11) if and only if

$$E_t x_{t+1} \cdot s \cdot k_{t+1}^{\alpha} \geq \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} \cdot \left(x_t + x_t^{NP} + x_t^{NU}\right) \cdot s \cdot k_t^{\alpha} = \frac{\left(x_t + x_t^{NP} + x_t^{NU}\right) \cdot \delta}{A + (1-\delta) \cdot x_t^{NP} - \delta \cdot x_t} \cdot \alpha \cdot k_{t+1}^{\alpha}, \quad (13)$$

for all t. The LHS of Condition (13) is $E_t \left\{I_{t+1} - D_{t+1}\right\}$ while the RHS is $E_t \left\{R_{t+1} \cdot I_t\right\}$. A chain of investments can satisfy this condition if and only if

$$\alpha < \begin{cases} s \cdot \dfrac{A}{\delta} & \text{if } A > 1-\varepsilon \\ s \cdot \dfrac{A}{\delta} \cdot \max\left\{1, \dfrac{1}{4 \cdot (1-\varepsilon) \cdot A}\right\} & \text{if } A \leq 1-\varepsilon \end{cases}.$$

Otherwise x_t would have to grow continuously and eventually exceed one, which is not possible. But this is the condition for the existence of bubbles in Proposition 1. It is also the condition for being in regions II-IV of Figure 3. Bubble creation thus makes the bubbly episodes of region IV possible. In these episodes, bubbles lower the rate of return making the chains of investments they replace dynamically inefficient.

We end this discussion by noting that the special case in which investments are homogenous and financial frictions are irrelevant, i.e. $\delta = 1$, exhibits an interesting property: if there exists a dynamically inefficient chain of investments, then the chain of all investments must also be dynamically inefficient. This is because all investments are homogenous.[20] Thus, the condition for bubbly episodes to exist implies that aggregate investment exceeds capital income, i.e. $\alpha < s$. Abel et al. (1989) used this result to argue that rational bubbly episodes are not possible in industrial economies, since in all of them aggregate investment falls short of capital income.

This argument is not quite right, though, since it is based on the dubious assumption that financial frictions do not matter in industrial economies. If $\delta < 1$, observing that $\alpha > s$ does not rule out rational bubbly episodes. This is for two reasons: (i) if $s < \alpha < s \cdot \frac{A}{\delta}$ (regions II and III), in the fundamental state there are dynamically inefficient chains of investments; and (ii) if $s \cdot \frac{A}{\delta} \leq \alpha < s \cdot \frac{A}{\delta} \cdot \frac{1}{4 \cdot (1-\varepsilon) \cdot A}$ (region IV), there are no dynamically inefficient chains of

[20] Once again, we note that the chain that contains all investments in the economy is dynamically inefficient in equilibria in which bubbles *do not* replace all investments. There exists no equilibrium in which all investments are replaced by bubbles.

investment in the fundamental state but expansionary bubbly episodes that lower the return to investment would create such chains themselves.

3.3 Shocks to investor sentiment as a source of business cycles

We study next the macroeconomic effects of bubbly episodes. To do this, rewrite the law of motion of the capital stock using the definition of x_t and x_t^{NP}:

$$k_{t+1} = \begin{cases} \left[A + (1-\delta) \cdot x_t^{NP} - \delta \cdot x_t\right] \cdot s \cdot k_t^{\alpha} & \text{if } \dfrac{x_t + x_t^{NP}}{1-\varepsilon} < 1 \\ (1 - x_t) \cdot s \cdot k_t^{\alpha} & \text{if } \dfrac{x_t + x_t^{NP}}{1-\varepsilon} \geq 1 \end{cases} . \tag{14}$$

Equation (14) describes the dynamics of the capital stock for any admissible stochastic process for the bubble $\left\{x_t, x_t^{NP}, x_t^{NU}\right\}$ satisfying Equations (9) and (10). Interestingly, bubbly episodes can be literally interpreted as shocks to the law of motion of the capital stock. These shocks do not reflect any fundamental change in preferences and technology. Instead, they can aptly be described as shocks to investor sentiment.

These shocks also have independent effects on consumption and therefore welfare, which in this model happen to be exactly the same:

$$c_t = (\alpha + x_t \cdot s) \cdot k_t^{\alpha}. \tag{15}$$

Equation (15) shows how bubbles affect consumption through two channels. First, contemporary bubbles increase consumption by raising the share of output in the hands of the old. This first effect is the same for all bubbly episodes, regardless of their type. Second, past bubbles affect consumption through their effect on the contemporary capital stock. This second effect clearly depends on the type of bubbly episode. In contractionary bubbly episodes, it lowers the capital stock and consumption. In expansionary bubbly episodes, it raises the capital stock and consumption.

To illustrate the potential of investor sentiment shocks for business cycle theory, we use next a simple example. Let $z_t \in \{F, B\}$ be the state or regime of the economy, with $\Pr\left[z_{t+1} = B/z_t = F\right] = q$ and $\Pr\left[z_{t+1} = F/z_t = B\right] = p$ for all t. That is, the economy switches between the fundamental state and bubbly episodes with transition probabilities q and p. When the economy is in the fundamental state, i.e. $z_t = F$, we have that $x_t^{NP} = x_t^{NU} = x_t = 0$. When the economy is in a bubbly episode, i.e. $z_t = B$, we have that:

$$x_t^{NP} = \eta_0 + \eta_1 \cdot x_t, \;\; x_t^{NU} = 0, \quad \text{and} \;\; x_t = \frac{\alpha}{s \cdot (1-p)} \cdot \frac{\delta \cdot [(1+\eta_1) \cdot x_t + \eta_0]}{A + (1-\delta) \cdot \eta_0 + [(1-\delta) \cdot \eta_1 - \delta] \cdot x_t} + u_t \tag{16}$$

where $u_t = \begin{cases} \sigma & \text{with prob. } 0.5 \\ -\sigma & \text{with prob. } 0.5 \end{cases}$. Thus, $E_t u_{t+1} = 0$ and $E_t u_{t+1}^2 = \sigma^2$. We choose parameters such that bubbly episodes never exceed the savings of the unproductive investor, i.e. $\frac{x_t + x_t^{NP}}{1-\varepsilon} < 1$; and bubbly episodes are always expansionary, i.e. $x_t^{NP} > x_t \cdot \frac{\delta}{1-\delta}$. Equation (16) describes a specific stochastic process for investor sentiment shocks. In applications, one would choose the process that best describes the empirical behavior of the bubble component of asset prices, just as we typically choose processes for productivity shocks that best describe the empirical behavior of total factor productivity.[21]

Figure 4 shows the result of simulating the economy using this example.[22] The figure plots output (k_t^{α}), consumption (c_t) and the bubble ($b_t + b_t^{NP}$) in each period. Initially, the economy begins with a low capital stock and grows towards the fundamental steady state. In period 10, there is a shock to investor sentiment that fuels a bubbly episode and raises the efficiency of investment. Consequently, the law of motion of the capital stock shifts upwards and the economy starts transitioning towards a higher, bubbly, steady state. Output and consumption increase, although they fluctuate throughout the bubbly episode along with the bubble. In period 38, a shock to investor sentiment ends this first episode and the economy suffers a sharp contraction. Output and consumption collapse and they stabilize around the fundamental steady state. Ten periods later, there is another shock to investor sentiment that starts a second bubbly episode and the economy expands again. This economy therefore experiences a business cycle that is driven solely by investor sentiment shocks. Despite its simplicity, this example shows that introducing these shocks into quantitative business-cycle models is a promising strategy to account for the stylized facts mentioned in the introduction.

[21] Indeed, there is a strong parallelism between technology shocks and investor sentiment shocks, since their data counterparts are both obtained as residuals. Growth in total factor productivity is the difference between growth in output and growth in factor usage. Growth in the bubble is the difference between the growth of asset prices and growth in the net present value of their payoffs. See Carvalho et al. (2011).

[22] To produce this figure, we assume that $p = 0.11$, $q = 0.11$, $\eta_0 = 0.15$, $\eta_1 = 0.18$, $\delta = 0.1$, $\sigma = 0.035$, $\varepsilon = 0.02$ and $\alpha = 0.4$.

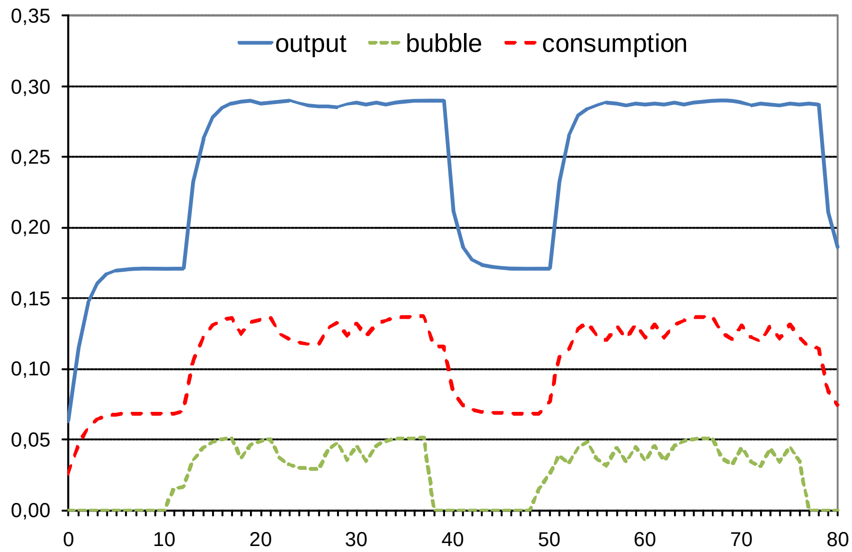

Figure 4

In Figure 4, the effects of bubbly episodes are transitory because the economy is stationary. This is due to diminishing returns, i.e. capital accumulation makes capital abundant and lowers its average product. Bubbly episodes would have long-lasting effects, however, if the economy were non-stationary. To illustrate this, in the appendix we generalize the production structure of the economy and allow for constant or increasing returns to capital accumulation. In particular, we assume that the final good is produced by assembling a continuum of intermediate inputs. The presence of fixed costs then creates a market-size effect, i.e. capital accumulation increases the number of intermediate inputs and this raises the average product of capital. We find that output is $y_t = k_t^{\alpha \cdot \mu}$; where the parameters $\alpha < 1$ and $\mu > 1$ reflect these two opposing forces. If diminishing returns are strong and market-size effects are weak, i.e. $\alpha \cdot \mu < 1$, capital accumulation lowers the average product of capital and the economy is stationary. If instead diminishing returns are weak and market-size effects are strong, i.e. $\alpha \cdot \mu \geq 1$, capital accumulation raises the average product of capital and the economy is non-stationary. Interestingly, this generalization does not affect the conditions for the existence of bubbly episodes in Propositions 1 and 2.[23] It does however make it possible for transitory bubbly episodes to have permanent effects. We illustrate this in the appendix with the help of an example in which a bubbly episode takes the economy out of a negative-growth trap. Even though the bubbly episode ends, the path of the economy has changed forever.

[23]Moreover, it has only minor effects on the formal structure of the model: Equations (9) and (10) remain the same while, in Equation (14), the exponents of k_t become $\alpha \cdot \mu$ instead of α.

4 Further issues and research agenda

We have developed a stylized model of economic growth with bubbles. In this model, financial frictions lead to equilibrium dispersion in the rates of return to investment. During bubbly episodes, unproductive investors demand bubbles while productive investors supply them. Thus, bubbly episodes channel resources towards productive investment raising the growth rates of capital and output. The model also illustrates that the existence of bubbly episodes requires some investment to be dynamically inefficient: otherwise, there would be no demand for bubbles. This dynamic inefficiency, however, might be generated by an expansionary bubble itself.

The model shows that bubbles implement a set of intra- and inter-generational transfers that can potentially have strong macroeconomic effects. In our stylized model, these transfers happen exclusively in the market for bubbles. But this need not be so. Martin and Ventura (2011) introduce a credit market in which investors are subject to borrowing constraints. The prospect of a future bubble raises the collateral of productive investors and allows them to borrow and invest more. In this setup, bubbles also help transfer resources from unproductive to productive investors in the credit market. Ventura (2011) introduces a distinction between consumption and investment goods. Bubbles reduce unproductive investments and lower the price of investment goods, allowing productive investors to invest more. In this setup, bubbles transfer resources from unproductive to productive investors in the goods market. These are only suggestive examples. Much more needs to be done to understand the role of bubbles in resource allocation.

The model also lays the theoretical groundwork for introducing investor sentiment shocks into applied macroeconomic models. In ongoing work, Carvalho et al. (2011), we develop a quantitative model of the financial accelerator with asset bubbles. This quantitative model contains a much more sophisticated and realistic description of preferences and demography, and cannot be analyzed with the simple analytical methods we have used here. Business cycles are driven by two types of shocks: fundamental shocks that affect technology and preferences; and investor sentiment shocks that lead to the appearance and collapse of bubbles in financial markets. Our immediate goal is to calibrate the model with data from industrialized economies and use it to explore the relative importance of both types of shocks in recent macroeconomic history.

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5 Appendix: the model with endogenous growth

Assume that the production of the final good consists of assembling a continuum of intermediate inputs, indexed by $m \in [0, m_t]$ according to a symmetric CES function:

$$y_t = \lambda \cdot \left(\int_0^{m_t} q_{tm}^{\frac{1}{\mu}} \cdot dm \right)^{\mu}, \tag{17}$$

where q_{tm} denotes units of the variety m of intermediate inputs and $\mu > 1$. The constant $\lambda = (\mu)^{-\mu} \cdot (1-\mu)^{\mu-1}$ is a normalization parameter. Final-good producers are competitive. Production of intermediate inputs entails variable and fixed costs:

$$q_{tm} = (l_{tm,v})^{1-\alpha} \cdot (k_{tm,v})^{a}, \tag{18}$$

$$f_{tm} = \begin{cases} 1 = (l_{tm,f})^{1-\alpha} \cdot (k_{tm,f})^{a} & \text{if } q_{tm} > 0 \\ 0 & \text{if } q_{tm} = 0 \end{cases}, \tag{19}$$

where f_{tm} is the fixed cost and $l_{tm,v,}$ $l_{tm,f}$, $k_{tm,v}$ and $k_{tm,f}$ are the labor and capital, variable and fixed costs of producing variety m. Input varieties become obsolete in one generation and, as a result, all generations must incur the fixed costs. The production of intermediate inputs takes place under under monopolistic competition and free entry.

This production structure is a special case of that considered by Ventura (2005). He shows that, under the assumptions made, the output of the economy is given by $y_t = k_t^{\alpha\cdot\mu}$, whereas competition in factor markets implies that $w_t = (1-\alpha) \cdot k_t^{\alpha\cdot\mu}$ and $r_t = \alpha \cdot k_t^{\alpha\cdot\mu-1}$.We can generalize Equations (6), (7) and (8) as follows:

$$E_t \left\{ \frac{b_{t+1}}{b_t + b_t^{NP} + b_t^{NU}} \right\} \begin{cases} = \delta \cdot \alpha \cdot k_t^{\alpha\cdot\mu-1} & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s \cdot k_t^{\alpha\cdot\mu}} < 1 \\ \in \left[\delta \cdot \alpha \cdot k_t^{\alpha\cdot\mu-1}, \alpha \cdot k_t^{\alpha\cdot\mu-1} \right] & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s \cdot k_t^{\alpha\cdot\mu}} = 1 \\ = \alpha \cdot k_t^{\alpha\cdot\mu-1} & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s \cdot k_t^{\alpha\cdot\mu}} > 1 \end{cases}, \tag{20}$$

$$0 \leq b_t \leq s \cdot k_t^{\alpha\cdot\mu}, \tag{21}$$

$$k_{t+1} = \begin{cases} s \cdot A \cdot k_t^{\alpha\cdot\mu} + (1-\delta) \cdot b_t^{NP} - \delta \cdot b_t & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s \cdot k_t^{\alpha\cdot\mu}} < 1 \\ s \cdot k_t^{\alpha\cdot\mu} - b_t & \text{if } \dfrac{b_t + b_t^{NP}}{(1-\varepsilon)\cdot s \cdot k_t^{\alpha\cdot\mu}} \geq 1 \end{cases}, \tag{22}$$

The only difference with the model in the main text lies in the exponents of k_t, which are now $\alpha \cdot \mu$ instead of α. For a given initial capital stock and bubble, $k_0 > 0$ and $b_0 \geq 0$, a competitive equilibrium is a sequence $\left\{k_t, b_t, b_t^{NP}, b_t^{NU}\right\}_{t=0}^{\infty}$ satisfying Equations (20), (21) and (22). It is easy to verify that this generalization of the production structure does not affect the conditions for the existence of bubbly episodes, as Propositions 1 and 2 apply for any value of μ. To see this, re-define $x_t \equiv \frac{b_t}{s \cdot k_t^{\alpha \cdot \mu}}$, $x_t^{NP} \equiv \frac{b_t^{NP}}{s \cdot k_t^{\alpha \cdot \mu}}$ and $x_t^{NU} \equiv \frac{b_t^{NU}}{s \cdot k_t^{\alpha \cdot \mu}}$, and check that Equations (9) and (10) still apply.

When $\alpha \cdot \mu \geq 1$, even transitory bubbly episodes can have permanent effects on the levels and growth rates of capital and output. Figure A1 illustrates this. The left panel depicts the case of an expansionary bubble. Initially, the economy is in the fundamental state and the appropriate law of motion is k_{t+1}^F. Initially, the capital stock is below the fundamental steady state, i.e. $k_t < k^F \equiv (s \cdot A)^{\frac{1}{1-\alpha \cdot \mu}}$, and growth is negative. This economy is caught in a "negative-growth trap". When an expansionary bubble pops up, it reduces unproductive investments and uses part of these resources to increase productive investments. During the bubbly episode, the law of motion of capital lies above k_{t+1}^F: in the figure, k_{t+1}^B represents the initial law of motion when the episode begins. Throughout the episode, k_{t+1}^B may shift as the bubble grows or shrinks. Growth may be positive if, during the bubbly episode, the capital stock lies above its steady-state value. Eventually, the bubble bursts but the economy might keep on growing if the capital stock at the time of bursting exceeds k^F. The bubbly episode, though temporary, leads the economy out of the negative-growth trap and it has a permanent effect on long-run growth. Naturally, it is also possible for contractionary bubbles to lead the economy into a negative growth trap thereby having permanent negative effects on growth: the right panel of Figure A1 shows this possibility.

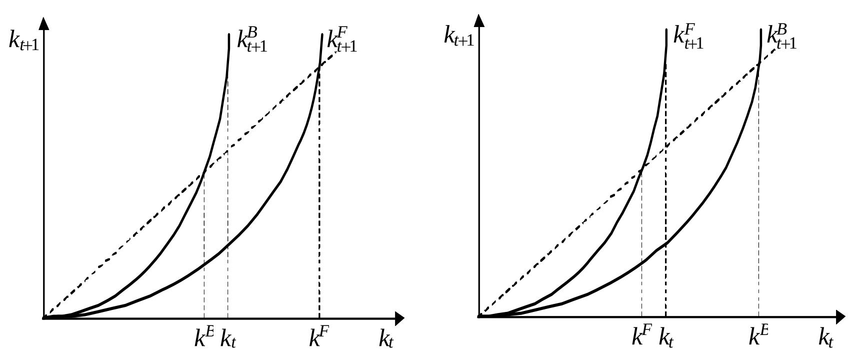

Figure A1